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# Pure quantum solutions of bohmian quantum gravity

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Abstract: In this paper we have investigated the *pure quantum* solutions of Bohmian quantum gravity. By pure quantum solution we mean a solution in which the quantum potential cannot be ignored with respect to the classical potential, especially in Bohmian quantum gravity we are interested in the case where these two potentials are equal in their magnitude and in fact their sum is zero. Such a solutions are obtained both using the perturbation and using the linear field approximation.

Keywords: Models of Quantum Gravity, Cosmology of Theories beyond the SM.

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## Contents





## 1. Introduction

The idea of quantization of the gravitational field is supported by different reasons. Quantum gravity is a good candidate for a fundamental theory because it is expected to avoid the general relativity singularities. This theory must lead us to some understanding of the initial conditions of the universe, inflation epoch, and so on. Furthermore, if one looks for the unification of gravity with the other fields, the fundamental description of gravity should be achieved by its quantization, just like the other fields.

Many approaches exist for quantization of gravity. One of them is the canonical quantization. Depending on the choice of the canonical variables, it can be divided into many classes. One of them is the geometrodynamical approach, which uses ADM decomposition. In this decomposition, dynamical variables are the three dimensional metric ($h_{ij}(x)$) having six independent elements. A specific character of quantum gravity is the existence of constraint, resulted from the four dimensional diffeomorphism invariance. Quantum gravity hamiltonian is given by:

$$\widehat{H} = \int d^3x \left( N^{\perp}\widehat{\mathcal{H}}^{\perp} + N^i \widehat{\mathcal{H}}_i \right), \tag{1.1}$$

where $N^{\perp}$ and $N^i$ are the lapse and shift functions respectively. These are the remaining non-dynamical four elements of the space-time metric. $\widehat{\mathcal{H}}^{\perp}$ and $\widehat{\mathcal{H}}_i$ are the time and three diffeomorphism invariance constraints. According to the Dirac's canonical quantization scheme, the constraints should kill the wavefunction of the universe. In terms of the canonical operators, we have the WDW equation:

$$\left[h^{-q}\frac{\delta}{\delta h_{ij}}h^{q}G_{ijkl}\frac{\delta}{\delta h_{kl}}+\sqrt{h}^{(3)}\mathcal{R}+\frac{1}{2\sqrt{h}}\frac{\delta^2}{\delta\phi^2}-\frac{1}{2}\sqrt{h}h^{ij}\partial_i\phi\partial_j\phi-\frac{1}{2}\sqrt{h}V(\phi)\right]\Psi=0 \tag{1.2}$$

and the momentum constraint:

$$i\left[2\nabla_j\frac{\delta}{\delta h_{ij}}-h^{ij}\partial_j\phi\frac{\delta}{\delta\phi}\right]\Psi=0\,, \tag{1.3}$$

where $\phi$ denotes the matter field, $G_{ijkl}$ is the DeWitt metric with signature $(-++++ +)$, $q$ is the ordering parameter, and $h=\det(h_{ij})$.

Although in this manner, the canonical quantization is applied to gravity, but in general the application of the standard quantum mechanics to gravity leads to many difficulties. Some of them are conceptual and some others are related to the mathematical structure of quantum mechanics. Because of the above problems, some physicists are interested in alternatives to quantum mechanics. Among these alternatives, de Broglie-Bohm theory [1] is of special importance. This theory has some special characteristics, such as:

- It provides an explanation of the individual events. In addition, its statistical content is just like the standard quantum mechanics.
- It presents an objectively real point of view of nature. The system exists independent of whether we measure it or not. So, it is not merely an abstract description of the system.
- The outcome of any measurement process is described by a causally related series of individual processes.
- All the particles have a well defined trajectory in the space-time, which can be evaluated from the quantum Hamilton-Jacobi function and the initial conditions.
- The wavefunction plays two roles. Its norm leads to the probability distribution, whereas its phase is proportional to the quantum Hamilton-Jacobi function.
- The guidance equation in this theory leads to the time evolution of dynamical variables independent of whether the wavefunction depends or not depends on time.
- de Broglie-Bohm theory is one of the appropriate theories of the universe, because it can be applied to the world as a whole, without any division to observer and observant.

In the de Broglie-Bohm theory the quantum Hamilton-Jacobi function satisfies some Hamilton-Jacobi equation modified by the quantum potential:

$$\frac{\partial S}{\partial t}+\frac{|\vec{\nabla} S|^2}{2m}+V+\mathcal{Q}=0\,, \tag{1.4}$$

where the quantum potential is given by:

$$\mathcal{Q}=-\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{1.5}$$

and $S$ is the Hamilton-Jacobi function, and $\rho$ is the ensemble density. Furthermore, in order to preserve the probability conservation, the continuity equation should be satisfied:

$$\frac{\partial \rho}{\partial t}+\vec{\nabla}\cdot\left(\rho\frac{\vec{\nabla} S}{m}\right)=0\,. \tag{1.6}$$

The equations (1.4) and (1.6) can be composed in such a way that the Schrödinger equation is resulted (setting $\Psi=\sqrt{\rho}\exp(iS/\hbar)$). But it must be noted that the physical content of the de Broglie-Bohm and the standard quantum mechanics are completely different. This can be seen easily from some of the above mentioned characteristics of the de Broglie-Bohm theory. In the other words, the polar decomposition of the wavefunction is the simplest way of obtaining the quantum Hamilton-Jacobi and continuity equations. But the fact that one must include some quantum potential with that specific form, can be understood by physical intuition. In ref. [2] some arguments are presented that leads one to write the quantum Hamilton-Jacobi equation and the continuity eqation without any reference to the Schrödinger equation.

Note that in the quantum Hamilton-Jacobi equation, setting $\mathcal{Q}=0$ one gets the classical equations of motion. This means that the *classical limit* can be defined as the case in which both the quantum potential and the quantum force may be ignored with respect to the classical counterparts, i.e. $|\mathcal{Q}|\ll|V|$ and $|-\vec{\nabla}\mathcal{Q}|\ll|-\vec{\nabla}V|$. On the other hand one can define an opposite limit which can be called *the pure quantum limit* where neither quantum potential nor quantum force can be ignored with respect to the classical counterparts, i.e. $|\mathcal{Q}|\gtrsim|V|$ and $|-\vec{\nabla}\mathcal{Q}|\gtrsim|-\vec{\nabla}V|$. In the classical limit the classical trajectory is slightly modified by quantum correction, while in the pure quantum limit one gets a trajectory that is not similar to any classical solution. Here we are interested in the case that the sum of the quantum and classical potentials are equal to the total energy of the system, i.e. $-\partial S/\partial t=V+\mathcal{Q}$. Thus from the quantum Hamilton-Jacobi equation, $\vec{\nabla}S=0$ and the system is at rest. So this is a stable situation of the system. It must be noted here that vanishing phase gradient is an important case in the non-relativistic de-Broglie-Bohm theory. For example, this occurs in the ground state of hydrogen atom or all the energy eigenstates of one dimensional harmonic oscillator.



JHEP05(2001)037

By expressing the wavefunction of the universe in the polar form, the equations (1.2) and (1.3) lead to:

$$G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}}+\frac{1}{2\sqrt{h}}\left(\frac{\delta S}{\delta\phi}\right)^2-\sqrt{h}\left({}^{(3)}\mathcal{R}-\mathcal{Q}_G\right)+\frac{\sqrt{h}}{2}h^{ij}\partial_i\phi\partial_j\phi+$$
$$+\frac{\sqrt{h}}{2}\left(V(\phi)+\mathcal{Q}_M\right)=0\,, \quad (1.7)$$

where the gravity quantum potential is given by:

$$\mathcal{Q}_G=-\frac{1}{\sqrt{h}\sqrt{\rho}}\left(G_{ijkl}\frac{\delta^2\sqrt{\rho}}{\delta h_{ij}\delta h_{kl}}+h^{-q}\frac{\delta h^q G_{ijkl}}{\delta h_{ij}}\frac{\delta\sqrt{\rho}}{\delta h_{kl}}\right) \quad (1.8)$$

and the matter quantum potential is defined as:

$$\mathcal{Q}_M=-\frac{1}{h\sqrt{\rho}}\frac{\delta^2\sqrt{\rho}}{\delta\phi^2} \quad (1.9)$$

and the continuity equation:

$$\frac{\delta}{\delta h_{ij}}\left[2h^q G_{ijkl}\frac{\delta S}{\delta h_{kl}}\rho\right]+\frac{\delta}{\delta\phi}\left[\frac{h^q}{\sqrt{h}}\frac{\delta S}{\delta\phi}\rho\right]=0\,. \quad (1.10)$$

The momentum constraint leads to:

$$2\nabla_j\frac{\delta\sqrt{\rho}}{\delta h_{ij}}-h^{ij}\partial_j\phi\frac{\delta\sqrt{\rho}}{\delta\phi}=0\,; \qquad 2\nabla_j\frac{\delta S}{\delta h_{ij}}-h^{ij}\partial_j\phi\frac{\delta S}{\delta\phi}=0\,. \quad (1.11)$$

The guiding equations are:

$$\frac{\delta S}{\delta h_{kl}}=\pi^{kl}\equiv\sqrt{h}\left(K^{kl}-h^{kl}K\right); \qquad \frac{\delta S}{\delta\phi}=\pi_\phi\equiv\frac{\sqrt{h}}{N^\perp}\dot{\phi}-\sqrt{h}\frac{N^i}{N^\perp}\partial_i\phi\,, \quad (1.12)$$

where $K^{ij}$ is the extrinsic curvature. Since in the WDW equation, the wavefunction is in the ground state with zero energy, then the stability condition of the metric and of the matter field is:

$$h^{ij}\partial_i\phi\partial_j\phi+V(\phi)-2{}^{(3)}\mathcal{R}+\mathcal{Q}_M+2\mathcal{Q}_G=0 \quad (1.13)$$

which is a pure quantum solution. This equation can simply derived from the equation (1.7) by setting all functional derivatives of $S$ equal to zero. In this paper we first solve the pure quantum case perturbatively and then investigate the pure quantum solution without matter in the linear field approximation.

## 2. Perturbative solution of pure quantum cases

First we shall discuss the pure quantum solutions using the method of perturbation in solving the Bohmian quantum gravity equations. Recently [3] a perturbative method of solving the classical Hamilton-Jacobi equations of general relativity has been presented. In ref. [4], this approach is adopted to solving Bohmian quantum gravity equations. In this method the quantum Hamilton-Jacobi function and the norm of the wavefunction have been expanded in terms of powers of spatial gradiants of the metric and matter fields. (the so called long wavelength approximation) Solutions up to the second order are derived. At each order the form of $S$ and $\sqrt{\rho}$ functionals is chosen in such a way that they are 3 diffeomorphism invariants. According to [4], we set:

$$\sqrt{\rho} = \exp(\Omega)\,; \qquad \Omega = \sum_{n=0}^{\infty} \Omega^{(2n)}\,. \tag{2.1}$$

In the pure quantum case which we are interested in here, the continuity equation will be satisfied trivially. Therefore at each order it is sufficient to solve quantum Hamilton-Jacobi equation. So we must set:

$$-\sqrt{h}\left({}^{(3)}\mathcal{R} - \mathcal{Q}_G\right) + \frac{1}{2}\sqrt{h}\partial_i\phi\partial^i\phi + \frac{1}{2}\sqrt{h}\,(V + \mathcal{Q}_M) = 0\,. \tag{2.2}$$

### 2.1 Zeroth order solution

In this order the quantum Hamilton-Jacobi equation is:

$$-2\sqrt{h}G_{ijkl}\left(\frac{\delta^2\Omega^{(0)}}{\delta h_{ij}\delta h_{kl}} + \frac{\delta\Omega^{(0)}}{\delta h_{ij}}\frac{\delta\Omega^{(0)}}{\delta h_{kl}}\right) + (q+3/2)h_{ij}\frac{\delta\Omega^{(0)}}{\delta h_{ij}} - \frac{\delta^2\Omega^{(0)}}{\delta\phi^2} - \\ - \left(\frac{\delta\Omega^{(0)}}{\delta\phi}\right)^2 + hV(\phi) = 0\,. \tag{2.3}$$

The appropriate choice of $\Omega^{(0)}$ which is 3 diffeomorphism invariant is:

$$\Omega^{(0)} = \int d^3x\ \sqrt{h}J(\phi)\,. \tag{2.4}$$

Substituting this in the relation (2.3), we have:

$$J'' - \frac{3}{2}(q+5)J - \sqrt{h}\left(\frac{3}{4}J^2 - J'^2\right) - \sqrt{h}V(\phi) = 0\,. \tag{2.5}$$

Since the terms containing metric must cancel each other, we have the two following equations:

$$J'' - \frac{3}{2}(q+5)J = 0\,; \qquad \frac{3}{4}J^2 - J'^2 + V(\phi) = 0\,. \tag{2.6}$$

These two equations can be easily solved to obtain the $K$ function and fix the form of $V$. We choose the solution as:

$$J = J_0\exp(\alpha\phi)\,; \qquad \alpha^2 = \frac{3}{2}(q+5) \tag{2.7}$$

so that:

$$V(\phi) = \left(\alpha^2 - \frac{3}{4}\right) J_0^2 \exp(2\alpha\phi)\,. \tag{2.8}$$

Therefore we have:

$$\Omega^{(0)} = \int d^3x \sqrt{h} J_0 \exp(\alpha\phi)\,. \tag{2.9}$$

### 2.2 Second order solution

In the second order, the quantum Hamilton-Jacobi equation for the pure quantum case is:

$$-\sqrt{h} G_{ijkl}\left(\frac{\delta^2\Omega^{(2)}}{\delta h_{ij}\delta h_{kl}} + 2\frac{\delta\Omega^{(0)}}{\delta h_{ij}}\frac{\delta\Omega^{(2)}}{\delta h_{kl}}\right) + \frac{q+3}{2} h_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} - \frac{1}{2}\frac{\delta^2\Omega^{(2)}}{\delta\phi^2} - \\ -\frac{\delta\Omega^{(0)}}{\delta\phi}\frac{\delta\Omega^{(2)}}{\delta\phi} - 2h\left({}^{(3)}\mathcal{R} - \frac{1}{2}\nabla_i\phi\nabla^i\phi\right) = 0\,.$$

On using the zeroth order equation and considering terms with and without metric separately, one arrives at:

$$\begin{aligned} -\sqrt{h} G_{ijkl}\frac{\delta^2\Omega^{(2)}}{\delta h_{ij}\delta h_{kl}} + \frac{q+3}{2} h_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} - \frac{1}{2}\frac{\delta^2\Omega^{(2)}}{\delta\phi^2} &= 0 \\ J h_{kl}\frac{\delta\Omega^{(2)}}{\delta h_{kl}} - 2J'\frac{\delta\Omega^{(2)}}{\delta\phi} - 4\sqrt{h}\left({}^{(3)}\mathcal{R} - \frac{1}{2}|\nabla\phi|^2\right) &= 0\,. \end{aligned} \tag{2.10}$$

In order to solve these equations we use the conformal transformation presented in [3, 4]. We set $h_{ij} = F^2(\phi) f_{ij}$ and assuming

$$\Omega^{(2)} = \int d^3x \sqrt{f}\left[\widetilde{{}^{(3)}\mathcal{R}} L(\phi) + f^{ij} M(\phi)\partial_i\phi\partial_j\phi\right], \tag{2.11}$$

where a $\sim$ over any quantity means that it is calculated with respect to the $f_{ij}$ metric, and

$$\frac{\delta\Omega^{(2)}}{\delta\phi} = A[F(\phi)]\frac{\delta\Omega^{(2)}}{\delta F} \tag{2.12}$$

one gets the following equations:

$$AF' = \frac{F^2}{12}\,; \qquad \left(\frac{19}{12} - \frac{q}{2}\right) F = F'\frac{dA}{dF}\,; \qquad \left(\frac{JF}{2A} - 2J'\right) L' - 4F = 0$$

$$-\left(\frac{JF}{2A} - 2J'\right) M' - 16F'' + 8\frac{F'^2}{F} + 2F = 0; \quad -\left(\frac{JF}{2A} - 2J'\right) M - 8F' = 0$$

with the solution:

$$q = 3\,; \qquad \alpha^2 = 12\,; \qquad A = \frac{\alpha}{3} F\,; \qquad F = F_0 e^{\phi/4\alpha}$$

$$M = -\frac{2F_0}{J_0(3/2 - 2\alpha^2)} e^{-(\alpha - 1/4\alpha)\phi}\,; \quad L = -\frac{4F_0}{J_0(2\alpha^2 + 3/8\alpha^2 - 2)} e^{-(\alpha - 1/4\alpha)\phi}$$

writing $\Omega^{(2)}$ in terms of the original metric, we have:

$$\Omega^{(2)} = \int d^3x\sqrt{h}\frac{e^{-\alpha\phi}}{J_0(2\alpha^2 + 3/8\alpha^2 - 2)}\left[{}^{(3)}\mathcal{R} + \frac{12\alpha^2 - 11}{6 - 8\alpha^2}|\nabla\phi|^2\right]. \tag{2.13}$$

Up to this point, we have found the stable solutions of Bohmian quantum gravity up to the second order approximation. This method can be applied to higher order approximations, too.

As we have presented some arguments in [4], in de Broglie-Bohm theory, the classical limit and the long wavelength approximation are two different domains, in general. This is resulted from the nature of the quantum potential. So we have the freedom of choosing any arbitrary value of quantum potential in the long wavelength approximation. For example here the sum of the quantum and classical potentials is equal to zero.

## 3. Pure quantum solutions without matter

In the previous section dependence of the wavefunction on the metric and the matter field is derived for the stable case perturbatively. Here we attack the problem with another strategy. For more simplicity we neglect the matter field here, and accept the following form for $\Omega$:

$$\Omega = \int d^3x\sqrt{h}{}^{(3)}\mathcal{R}. \tag{3.1}$$

For simplicity we have chosen $\Omega$ containing second spatial gradiants of metric. We want to find the spatial metric for the pure quantum state of the universe. Setting $q = -3/2$ (which sets the second term in $\mathcal{Q}_G$ equal to zero, and thus simplifies the equations) and $\delta S/\delta h_{ij} = 0$ in equation (2.2), and ignoring the matter field we have:

$$-\sqrt{h}{}^{(3)}\mathcal{R} - \hbar^2 G_{ijkl}\frac{\delta^2\Omega}{\delta h_{ij}\delta h_{kl}} - \hbar^2 G_{ijkl}\frac{\delta\Omega}{\delta h_{ij}}\frac{\delta\Omega}{\delta h_{kl}} = 0. \tag{3.2}$$

Substituting $\Omega$ in the above relation, we have the following equation for the scalar curvature:

$$\begin{aligned}\hbar^2\left(\frac{{}^{(3)}\mathcal{R}}{8} + \frac{{}^{(3)}\mathcal{R}^2}{4} + \frac{\nabla^2{}^{(3)}\mathcal{R}}{2} - {}^{(3)}\mathcal{R}_{ij}{}^{(3)}\mathcal{R}^{ij} - \nabla_i\nabla_j{}^{(3)}\mathcal{R}^{ij}\right) - \\ -\sqrt{h}\left[{}^{(3)}\mathcal{R} + \hbar^2\left({}^{(3)}\mathcal{R}_{ij}{}^{(3)}\mathcal{R}^{ij} - \frac{{}^{(3)}\mathcal{R}^2}{8}\right)\right] = 0.\end{aligned} \tag{3.3}$$

This equation is a second order relation of the scalar curvature and Ricci tensor, including their spatial gradiants. For finding the solution we use linear approximation. Suppose the spatial metric be of the form $h_{ij} = \eta_{ij} + \epsilon_{ij}$. Up to first order in $\epsilon$ the equation (3.3) gives

$$\partial^2\epsilon - \partial_i\partial_j\epsilon^{ij} = 0 \tag{3.4}$$



JHEP05(2001)037

In order to solve the above equation, one can expand $\epsilon^{ij}$ in terms of the spherical harmonics:

$$\epsilon^{ij} = \sum_{l,m} \alpha_{lm}^{ij} f_l(r) Y_{lm}(\theta, \varphi) \tag{3.5}$$

and then substitute this in the equation (3.4). Finally we find:

$$\alpha_{lm}^{ij} = \begin{pmatrix} \alpha_{lm} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad f_l(r) = r^{-l(l+1)/2} \tag{3.6}$$

Therefore the space-time metric is:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 + \sum_{lm} \alpha_{lm} r^{-l(l+1)/2} Y_{lm}(\theta, \varphi) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}, \tag{3.7}$$

where we have chosen $N^{\perp} = 1$ and $N_i = 0$. Some mathematical manipulations result that $\partial_t$ is a killing vector leading to the stationarity of the space-time metric. Also $\partial_\varphi$ is another killing vector provided $m = 0$. Since the parameter $t$ is not appeared in the space-time metric, our coordinate frame is consistent with the killing vector $\partial_t$. In addition because $N^{\perp} = 1$ and $N_i = 0$, the components of the Ricci tensor are:

$$\mathcal{R}_{00} = 0; \qquad \mathcal{R}_{0i} = 0; \qquad \mathcal{R}_{ij} = {}^{(3)}\mathcal{R}_{ij}. \tag{3.8}$$

This means $\mathcal{R} = {}^{(3)}\mathcal{R}$. Using the above relations one can see that not all the components of the Ricci tensor are zero. For example $\mathcal{R}_{22} \neq 0$. Therefore the quantum effects, change the classical solution (flat space-time) to a curved space-time. This fact is shown in a completely different way previously [5].

## 4. Conclusion

Pure quantum solutions, i.e. the solutions in which the classical potential and the quantum potential are of the same order are important, because they represent cases which are not in any way similar to the classical solutions. They have not classical limit.

Pure quantum solutions are very important in the non-relativistic de-Broglie-Bohm theory. Many of stationary states, like the ground state of hydrogen atom, or all the energy eigenstates of one dimensional harmonic oscillator are pure quantum states in which system is at rest. Such a solution may be prepared in the laboratory as a state very different from classical mechanics.

For the quantum cosmology case there is a difference. Since the universe is prepared in some specific state and cannot be reprepared in another way by us. The universe is believed thave classical limit and thus cannot be a pure quantum state. But it may be argued that at some times (times very close to bigbang, say) the state might be pure quantum state.

In the case of quantum gravity, we can prepare systems in pure quantum states at least in principle. So the investigation of such states is not only a mathematical exercise in the de-Broglie-Bohm quantum gravity and Cosmology, but also it may be useful for (i) the initial times of the universe, (ii) some specially prepared gravitational systems.

In this paper we studied the problem of how to solve the Bohmian quantum gravity equations in the case of pure quantum limit. We saw that there is at least two ways. First, one can use the method of perturbation and the second way is not perturbative but is written in the linear field approximation.

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